AIRCRAFT SHAPES AND THEIR AERODYNAMICS FOR FLIGHT AT SUPERSONIC SPEEDS

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Summary—For the task of achieving a given flight range, and with some overall assumptions about the structural and propulsive elements and a simple set of supersonic aerodynamics, box sizes with certain relations between span and length are determined, into which a supersonic aircraft must fit. Various layouts are then discussed, with suitable types of flow which not only give the required performance but are also acceptable for engineering purposes. The yawed wing, the swept wing-fuselage combination, and the slender wing are shown to offer potential solutions, each of these designed to have the same type of flow throughout its flight range. Slender wing aircraft are considered in more detail and some theoretical and experimental results are given.

1. *Introduction*

As seen in retrospect, the first 50 years of aeronautical engineering have been dominated by one characteristic shape of the practical aeroplane on the one hand and Prandtl's boundary-layer and aerofoil theories on the other. With these two aspects, aerodynamic theory and aircraft design most happily complemented one another. It is now time to realize that the classical aircraft shape constitutes a highly restrictive class of bodies and that classical aerodynamics amounts to the study of one special case of general fluid flow only. The future of aeronautics, with the prospect of almost unlimited available power and attainable speeds, therefore, requires not only the extension of existing theories to include thermodynamic and real-gas effects; it also requires the introduction and the study of new characteristic shapes and new types of flow, which again should suit one another as perfectly as was true for the classical aircraft and classical aerodynamics.

To explain the procedure we adopt in arriving at such new shapes, now for supersonic flight speeds, we begin with a brief recapitulation of how this procedure can be applied to the classical aircraft, for the simplest task: that to achieve a given flight range. After that, we are going to discuss the changes brought about by new means of propulsion and by the new set of supersonic aerodynamics. This is followed by a discussion of some possible solutions, which include swept wing-fuselage combinations, yawed wings, and slender wings. Only the latter involve an entirely new type of flow and this is discussed in some detail.

It is gratefully acknowledged that many of the findings reported here are the results of a concerted research effort on a large scale, in which most British aircraft firms and research establishments have been engaged over the last 3 or 4 years. The responsibility for the presentation and the conclusions is, of course, the author's.

2. *Prolegomena Concerning the Classical Aircraft*

We consider the simplest and most obvious task which an aircraft is required to perform, namely, to fly from one place to another and so to achieve a given flight range. Hence, according to Bréguet, a certain value of

$$\text{Range} = I - V_0 \times (L/D) \times \ln (W_1/W_f) \tag{1}$$

must be obtained. This equation contains a structural component—the ratio between the initial weight and the final weight, including fuel; an aerodynamic component—the ratio L/D between lift and drag at cruise; a speed term—the cruising speed V_0; and a propulsion term—taken here as the specific impulse, I, of the powerplant; I is inversely proportional to the specific fuel consumption. It is evident from Bréguet's equation alone that aircraft design is a compromise between structural, propulsive, and aerodynamic ingredients. However, our contention is that, with only a rudimentary knowledge of the structural and propulsive components, the aerodynamic component alone is sufficient to determine the general layout of the vehicle in the cases considered.

We now introduce some restrictions and stipulate that the aircraft should fly at subsonic speeds and, further, that it should be so designed that the means for providing stowage volume, lift, and propulsion are separate. If propulsion is to be obtained by propellers driven by piston engines, then we are concerned with what are basically constant-horsepower engines so that for each individual engine the product of I and V_0 remains roughly constant, not I itself. Again, for the whole family of piston-propeller engines, a higher required thrust at some higher flight speed implies a higher specific fuel consumption and a lower specific impulse so that, to a first order, the value of $I \times V_0$ may be taken as lying within a certain band, independent of the flight speed. On the structural side, it is sufficient to assume it to be known that no drastic changes in the weight factor occur with flight speed so that the value of W/W can be taken to lie within a certain band again, somewhere between 1 and 2, say. We are then left with the aerodynamic side where it is required that certain values of the lift-drag ratio be achieved; these may be taken as lying within a band between 15 and 25, for so-called long ranges.

We now proceed to show that such an aircraft, which consists of a separate volume-carrying fuselage, separate engines, and a separate lifting surface, and which is to achieve the required values of the lift-drag ratio, must have some characteristic dimensions, namely, an aspect ratio $A = 4s^2/S$ of the order 10, without invoking any detailed knowledge of its actual shape.

The drag of the aircraft will consist in part of skin friction forces along its surfaces and a slight form drag; these can be taken together in a coefficient C_{DF} which may be assumed to be independent of the lift force to a first order. $C_{DF} = 0{\cdot}01$ is a typically good value. The other part will be related to the mechanism by which the lift force is produced. Recalling physical principles first stated by Prandtl and Munk, we may say that the lift force must be associated with a downward momentum of the air behind the wing, and this in turn leads to a loss of kinetic energy and thus a drag force, the "vortex drag", which may be written as

$$C_{DL} = \frac{S}{4\pi s^2} K C_L^2 = \frac{K}{\pi A} C_L^2 \tag{2}$$

where

$$\varrho V_0 \frac{\pi s^2}{K}$$

is the virtual mass of air moved by the wing, with V_0 the forward speed and $2s$ the overall span of the wing. The as yet unknown factor K is of the order of unity for both planar and non-planar lifting systems.

With the overall drag of the form

$$C_C = C_{DF} + \frac{K}{\pi A} C_L^2 \tag{3}$$

the lift-drag ratio can be determined and comes out to have a maximum value

$$\left(\frac{L}{D}\right)_m = \sqrt{\frac{\pi A}{4KC_{DF}}} = \frac{1}{2}\frac{1}{\sqrt{C_{DF}C_{DL}/C_L^2}} \tag{4}$$

when

$$C_{Lm} = \sqrt{\frac{\pi A}{K} C_{DF}} = \sqrt{\frac{C_{DF}}{C_{DL}/C_L^2}} \tag{5}$$

i.e. when $C_{DF} = C_{DL}$. For the present purpose, and anticipating later steps in this procedure, it may be accepted that the best range is in fact not obtained when the aircraft is designed to fly at the C_L which gives the maximum value of L/D but that engine and weight considerations lead one to fly at a C_L-value slightly below that, typically at

$$C_L = \frac{1}{2}\sqrt{2}\, C_{Lm} = \frac{1}{2}\sqrt{2}\sqrt{\frac{\pi A}{K} C_{DF}} \tag{6}$$

in which case

$$\frac{L}{D} = \sqrt{\frac{8}{9}}\left(\frac{L}{D}\right)_m = \sqrt{\frac{2\pi A}{9K C_{DF}}} \tag{7}$$

Because it is immaterial to the present investigation which value of L/D is taken, it is this value of L/D that is considered in most numerical examples, unless otherwise stated.

For a required value of L/D, equation (7) may be regarded as determining the aspect ratio which is needed:

$$A = \frac{9}{2\pi} K C_{DF}\left(\frac{L}{D}\right)^2 \tag{8}$$

Typical values of A obtained in this way with $K = 1$ are:

Values of A (and of C_L)

for $L/D =$	15	20	25
when $C_{DF} = 0{\cdot}01$	3 (0·22)	6 (0·30)	9 (0·38)
when $C_{DF} = 0{\cdot}015$	5 (0·34)	9 (0·45)	13 (0·56)

On the whole, these values are of the order of 10 (i.e., not 1, nor 100), and even drastic changes in the values of K and C_{DF} cannot alter this basic result.*

Proceeding further with this hypothetical aerodynamic development of the classical aircraft, we now come to the aerodynamic design problem, namely, to investigate whether shapes can be found which have, in fact, the assumed physical properties and whether the type of flow implied is usable in engineering practice, i.e., whether it is steady and controllable and whether it can be maintained also in off-design conditions, preferably throughout the whole flight range, limiting our attention from now on to relatively thin wings, the spanwise extent of which is of an

* It should be borne in mind that equation (8) determines a "minimum aspect ratio for a given set of conditions and that the designer is at liberty to choose a higher aspect ratio. In that case, a higher payload may be carried provided that the saving in fuel due to the higher aspect ratio was greater than the cost in structure weight of achieving it. In the present context, this only strengthens the conclusion that classical aircraft are likely to have wings of moderate or large aspect ratio.

order greater than its streamwise extent, and to which a volume–providing fuselage and engines can be attached without any first-order interference. It will suffice here to recall that classical aerodynamics in the dual form of classical aerofoil theory and boundary-layer theory is admirably suited to deal with the wings of high aspect ratio with classical aerofoil sections that emerged, and fits the required conditions perfectly.

In particular, it is recalled that the shapes under consideration allow the assumption to be made that the flow over the wing is dominated by the characteristics of a two-dimensional flow in streamwise planes; and that the flow over the fuselage is basically that past an essentially non-lifting body of revolution. Further, there exists a two-dimensional type of flow in which viscous effects are confined to a thin boundary layer and wake. The body which gives rise to this flow possesses a sharp trailing edge so that the separation is confined and fixed to the trailing edge. Therefore, the trailing vorticity which is associated with the lift force is shed in the form of an essentially planar sheet. Also, it is possible to control this flow and its associated forces and moments so as to achieve a sufficient range of practical flight manoeuvres while keeping the separation at the wing trailing edge and, thus, maintaining essentially the same type of flow throughout. This type of flow also gives the best performance and handling qualities, compared with any other possible flow, and it has been generally agreed for 50 years that the greatest efforts should be made to maintain it. For a further discussion of the aerodynamic design problem, we refer to a paper by Maskell(1) and for an account of the relevant theories and observations in this light to a recent book by Thwaites(2).

In the present context, we note that within the aerodynamic system with the classical type of flow, equation (2) and the subsequent relations are very good approximations to the observed facts so that the procedure adopted is both consistent and realistic. Also, none of the often drastic developments in the fields of structural design, propellers and piston engines have changed the fundamental aerodynamic characteristics of the family of aircraft so obtained. Indeed, so successful was this family of aircraft that many of its features have come to be regarded as fundamental to any aircraft, whether supersonic, hypersonic, or of the VTO type, and, when jet engines made supersonic flight possible, it was assumed as a matter of course that a volume-providing non-lifting fuselage would have to be attached to a separate thin wing to do the lifting, with separate engines for propulsion. On the aerodynamic side, we thus proceeded automatically to investigate bodies of revolution and thin two-dimensional aerofoils at supersonic speeds and although it was found that their properties radically differed from those which, at low speeds, had led to the classical aircraft, missiles and aircraft of the same basic layout were, and still are, actively pursued. As a consequence, we had to live with what has been described as "lousy flows".

We believe that this development was a mistake and propose, therefore, to start again and investigate whether a procedure similar to that just explained but with different initial assumptions may lead us to more natural and more efficient solutions for flight at supersonic speeds.

3. *New Aerodynamic Requirements*

We still consider the simple task of achieving a given flight range, according to equation (1), but now at mainly supersonic speeds. There is no reason to suppose that the structural term will radically change by an order of magnitude. But the propulsion term differs fundamentally from that of the propeller-piston engine combination if we now consider jet engines that are the outcome of the one major development without which supersonic flight could not be contemplated. For jet-engine propulsion, the specific impulse, and the specific fuel consumption, stay roughly constant over a certain speed range for each individual engine as well as for the whole family of engines. They are basically "constant-thrust" rather than "constant-power" engines. Thus the value of I in equation (1) lies within a certain band, independent of the flight speed. As a consequence on the aerodynamic side, the product $V\times(L/D)$ or $R = M\times(L/D)$ is required to have certain values rather than L/D itself.

In principle, this aerodynamic requirement may be considered to apply through the whole Mach number range where air-breathing engines can be used, up to $M = 10$ perhaps. We must be prepared to find, however, that additional conditions, under which the given flight range is to be achieved, may be made such as those which may result from considerations of aerodynamic heating. Further, engine performance is likely to change with Mach number, to a second order, and while we can say that typical values of $R = ML/D$ lie between 15 and 30, a practical variation of the required value of L/D with Mach number may be less drastic. As a rough guide, values of $R = 4(M+3)$ may be regarded as "good" ones for long ranges of the transatlantic order up to $M = 5$ or 6, say. This gives

$L/D =$	14	10	8	7	6
for $M =$	1·2	2	3	4	6

Slightly lower values can probably be tolerated. The drop in the required value of L/D with Mach number has far-reaching consequences on the layout of the aircraft.

4. *A Set of Supersonic Aerodynamics*

Whereas we can see already that the reduction in the required value of L/D at supersonic speeds should lead to smaller values of the aspect ratio—A would be less than $1\frac{1}{2}$ at $M=2$ if equation (8) would still apply, with $K=1$ and $C_{DF}=0{\cdot}01$ — still another aspect demands our attention, namely, that a new set of aerodynamics is needed, because equation (8) and its basis can no longer be considered adequate. This must take account of the fact that wave drag terms appear, resulting from the volume and from the lift of the aircraft and depending primarily on the lengths of the volume and of the lifting surface. Most of what now follows will be explained by means of the simplest example where the length of the volume is the same as that of the lifting surface, in contrast to common practice. This is also the most realistic case because we shall find that it is very difficult, if not impossible, to make a volume-providing body non-lifting and to provide a lifting surface without volume. In this sense, most configurations cannot help being such that volume and lifting surface are "integrated".

A suitable set of geometric parameters is needed first. Apart from the wing area, S, three further parameters must account for the overall lenght, l, the overall span, $2s$, and the overall volume, V, of the wing or body, whatever its shape. Following Collingbourne, a consistent and convenient set is given by:

$$\frac{s}{l} \quad \text{the semispan-length ratio}$$

$$p=\frac{S}{2sl} \quad \text{a planform shape parameter} \tag{9}$$

$$\tau=\frac{V}{S^{3/2}} \quad \text{a volume parameter} \tag{10}$$

p is the ratio between the wing plan-area and the area of the circumscribed rectangle with the same span and length. $p={}^1/_2$ for wings of delta planform. Note that the aspect ratio is then

$$A=2\frac{s/l}{p} \tag{11}$$

It is not an independent parameter. A is increased by increasing the semispan-length ratio for constant p or by decreasing the planform shape parameter for constant s/l.

Four principal terms contribute to the overall drag of the aircraft in the set of aerodynamics considered here:

(1) Skin-friction drag along the surface of the aircraft, leading to the drag coefficient C_{DF} as before.

(2) Wave drag resulting from the volume of the aircraft. This can conveniently be defined as:

$$C_{DW} = \frac{128}{\pi}\frac{V^2}{Sl^4}K_0 = \frac{512}{\pi}\tau^2 p^2\left(\frac{s}{l}\right)^2 K_0 \tag{12}$$

where K_0 is a non-dimensional factor which can be determined by theory or experiment. Equation (12) takes account of the fact that V^2 and $1/l^4$ are the dominant factors for many configurations; the value of K_0 then depends primarily on the shape of the body enclosing the given overall volume. A value for K_0 of unity is typical for well-shaped bodies of revolution; in fact, $K_0 = 1$ corresponds to the Sears-Haack body.

(3) Wave drag resulting from the lift on the aircraft. This can be written in the form:

$$C_{DLW} = \frac{S}{2\pi}\beta^2\frac{C_L^2}{l^2}K_W = \frac{1}{\pi}C_L^2\, p\frac{s}{l}\beta^2 K_W \tag{13}$$

Again, it is convenient to isolate the dominant factors C_L^2, $1/l^2$ and β^2 for most configurations; the drag factor K_W then depends primarily on how the given overall lift is distributed over the surface at the given Mach number. For slender wings, $K_W = 1$ when the load is elliptically distributed along the length, corresponding to the well-known "lower bound" of R. T. Jones[(3)]. Thus the definition of K_W is consistent with that of K_0, both $K_0 = 1$ and $K_W = 1$ being obtained with the same kind of approximation.

(4) Vortex drag resulting from the lift on the aircraft. This can be left in the classical form, as in equation (2),

$$C_{DLV} = \frac{S}{4\pi}\frac{C_L^2}{s^2}K_V = \frac{1}{2\pi}C_L^2\frac{p}{s/l}K_V \tag{14}$$

because it is still convenient for many configurations which fly at supersonics speeds to isolate C_L^2 and $1/s^2$ as the dominant parameters. $K_V = 1$ is still obtained in inviscid flow for certain wings with planar vortex sheets when the load is elliptically distributed along the span.

We note here, and shall explain in some more details later, that values of unity for any of the drag factors K_0, K_W, and K_V must not be regarded as minimum values. It will be possible in all cases to obtain values below unity. $K_0 = K_W = K_V = 1$ only serve as convenient standard units.

In what follows, only these four drag terms will be used and the important assumption will be made that they are additive. Hence

$$C_D = C_{DF} + \frac{512}{\pi}\tau^2 p^2\left(\frac{s}{l}\right)^2 K_0 + \frac{1}{2\pi}C_L^2\frac{p}{s/l}\left[K_V + K_W 2\beta^2\left(\frac{s}{l}\right)^2\right] \tag{15}$$

which replaced equation (3) and has been derived in much the same way and with similar approximations. Adding the drag terms is a convenient and adequate approximation for many configurations; it is slightly less convenient for configurations where interference terms play a major part such as those where part of a volume-providing body is meant to interfere with a lifting surface or whether a propulsion unit is installed so as to interfere favourably with volume and lifting surface; it is also less convenient for configurations where essentially non-linear lift forces occur at cruise. However, equation (15) can still be used in all these cases provided care is taken in determining the values for the drag factors. For example, K_V may depend on C_L.

5. *Drag Forces and Overall Dimensions Obtained with this Set of Aerodynamics*

We can now determine some characteristic dimensions for all aircraft within this set of aerodynamics, without specifying detailed shapes and properties. To begin with, we realize that a compromise must be found between overall span and length of the aircraft, for a given wing area, because the drag tends to be very large when the span is too small (third term in equation (15)) and again when the length is too small (second and fourth terms in equation (15)). There must be a value of the ratio between span and length, i.e., of s/l for given p or of $\beta s/l$ for given M, at which C_D is smallest.

Comparing equation (15) with the earlier equation (3) for the classical aircraft, where the drag was the lower the higher the span, we now have s/l occurring both in the denominator (of the vortex drag term) and in the numerators (of the wave drag terms) so that we can state immediately that it will pay to use as low a value of p as possible and that a best "box size", s/l, will then follow, all this on aerodynamics grounds along without involving matters of structure or propulsion. This is demonstrated in Fig. 1*; it is an obvious consequence of the characteristic behaviour of the different drag terms. Curves for different C_L values are drawn in Fig. 2, which displays a region within which the lift-drag ratio L/D is highest. This value is of the required order. Also shown is a line

* To make the numerical values somewhat realistic, most values, unless otherwise stated, have been chosen to be typical of a large airliner to fly in the medium Mach number range, of $S = 6000$ ft.^{2} wing area and a volume parameter $\tau = 0{\cdot}04$, at between 35,000 ft. altitude at $M = 1$ and 75,000 ft. altitude at $M = 5$, with fully turbulent boundary layer, assuming the wanted area to be twice the wing planform area. In some cases, Figs. 1, 2, and 14, K_0 has been varied in a way approximating that of a particular thickness distribution ("Lord V"):

$$K_0 = 1{\cdot}17\,\frac{1+1{\cdot}5\,\beta s/l}{1+4\,\beta s/l}$$

The drag of this particular thickness distribution is considered by itself in Fig. 12.

$C_L = C_{Lm}/\sqrt{2}$, along which the C_L value is a given fraction of the value, C_{Lm}, at which the highest L/D is reached for $\beta s/l = \text{const.}$; as stated earlier, one might cruise near this line, for non-aerodynamic reasons.

Along such a line ($C_L = C_{Lm}/\sqrt{2}$), L/D reaches a maximum value for a certain s/l or $\beta s/l$, for a given value of τ. Similarly, values of τ can be

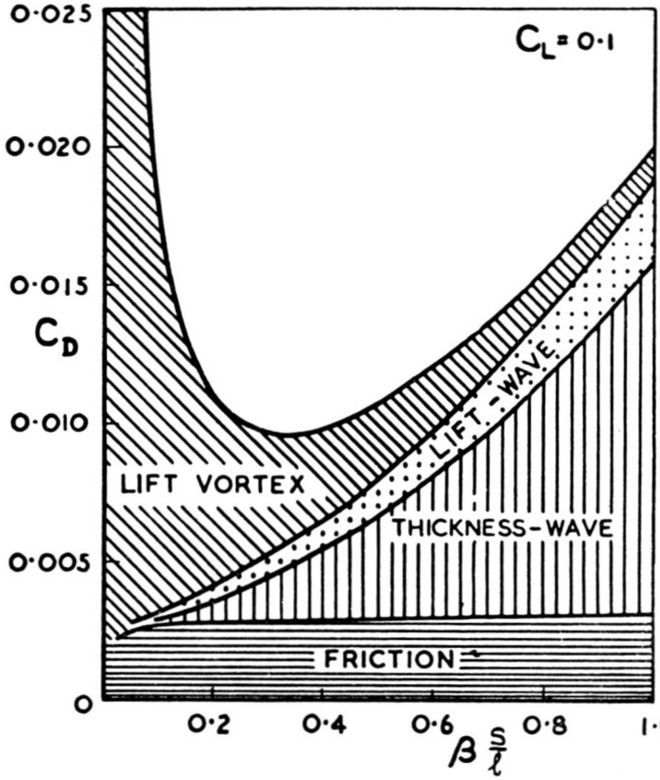

FIG. 1. Typical drag coefficients at $M = 2$ $p = 1/2$ $\beta = 0{\cdot}04$ $K_v = K_w = 1$ $S = 6000$ FT^2.

plotted along s/l, again along such lines, for required values of L/D and these curves show again maxima, i.e., thickest wings, for values of s/l, which are very close to those obtained before.

The main conclusion to be drawn from these results is that the best lift-drag ratios or thickest wings are obtained when $\beta s/l$ is well below unity—near 0·35 in this case where $M = 2$, so that s/l is near 0·2. We thus obtain a "box size", s/l into which the aircraft must fit. This varies with Mach number, and typical values are $s/l = 0{\cdot}4$ for $M = 1{\cdot}2$; $s/l = 0{\cdot}2$ for $M = 2$; $s/l = 0{\cdot}1$ for $M = 5$. So the box becomes narrower as the design Mach number is increased. As the main geometric characteristic, this box size assumes for supersonic configurations the part played by the aspect ratio for the classical aircraft.

As a result of many similar calculations we find that the best box size is not very sensitive to the actual values of the other parameters and drag factors, and that even drastic changes in the latter make very little differ-

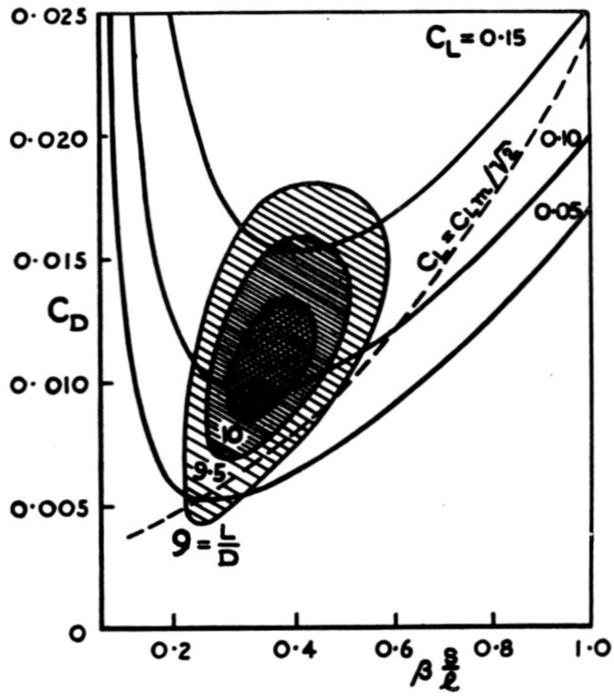

FIG. 2. Typical drag coefficients at $M = 2$ $p = 1/2$ $\beta = 0{\cdot}04$ $K_v = K_w = 1$ $S = 6000\ FT^2$.

ence. Smaller values of the planform parameter p correspond to slightly wider boxes; better (i.e. smaller) values of K_0 allow wider boxes; as do better values of K_W and worse values of K_V. Generally, worse (i.e. higher) values of K_0, K_W and K_V give lower L/D for a given τ or restrict τ to lower values for given L/D; whereas lower values of p improve matters. Further, if the wetted area is essentially greater than twice the planform area, then L/D is generally lower and best values occur at slightly higher values of s/l.

This general result is not substantially changed if configurations are considered where the length of the volume differs from that of the lifting surface, assuming for the moment that such configurations could be realized in practice. The extreme case of this kind is evidently obtained when only the last two terms in equation (15) are assumed to depend on s/l or, what is equivalent, when τ is assumed to be zero. In that case $\partial C_D/\partial(s/l) = 0$, i.e., the drag is lowest and L/D highest, when

$$\beta \frac{s}{l} = \sqrt{\frac{K_V}{2K_W}} \tag{16}$$

so that $\beta s/l \leqslant 0{\cdot}707$ since in most cases $K_W \geqslant K_V$. A small but non-zero thickness will bring the value of $\beta s/l$ for the best L/D well below 0·707.

We can, therefore, draw an important conclusion without knowing any details of actual configurations, namely, that the box size into which an aerodynamically efficient supersonic aircraft must fit is always less wide than long and that the box becomes the narrower the higher the Mach number. As a rough guide,

s/l lies between 0·3 and 0·4 and $\beta s/l$ is about 0·2 at $M = 1{\cdot}2$;
s/l lies between 0·15 and 0·25 and $\beta s/l$ is about 0·35 at $M = 2$;
s/l lies between 0·05 and 0·15 and $\beta s/l$ is about 0·5 at $M = 5$.

This means that the aircraft, whatever its detail shape, should always lie well within the Mach cone from its nose.

This result corresponds to that obtained earlier: that the classical aircraft normally has wings of moderate or large aspect ratio; and in the same way as that allowed us to restrict ourselves to what we call classical aerodynamics, we shall now attempt to use this new result to select such shapes which appear promising and cultivate their aerodynamics, and to leave others out altogether. At the same time, we shall try to follow the classical example in directing our attention towards the aerodynamics of such flows which physically exist and which are acceptable for engineering purposes. With this approach we differ from much of past practice in supersonic aerodynamics and design in two main respects: As a rule, thickness and lift effects have been treated and "optimized" separately; all too often, "optimum" layouts have been devised on the basis of theoretical flows to which nothing corresponded in nature.* When, as a result, the actual flows were "lousy", some of us resigned ourselves to this as an apparently unavoidable state of affairs and so a long string of patent-cures-in-a-small-way has appeared over the last 10 or 15 years.

6. *Some Possible Solutions*

Our next task is to fill with realistic aircraft shapes the boxes previously determined. Some of these are shown in Fig. 3, with some slight adjustments depending on the particular configuration used in anticipation of later results. As will be seen, we propose to deal with a swept wing-

* A typical example of this was given in Fig. 7 of R. T. Jones paper[4] at the last Congress.

fuselage combination, with a slender wing, and with a yawed wing as suggested 20 years ago by E. von Holst and taken up again by R. T. Jones[4] at the first Congress of this Council.

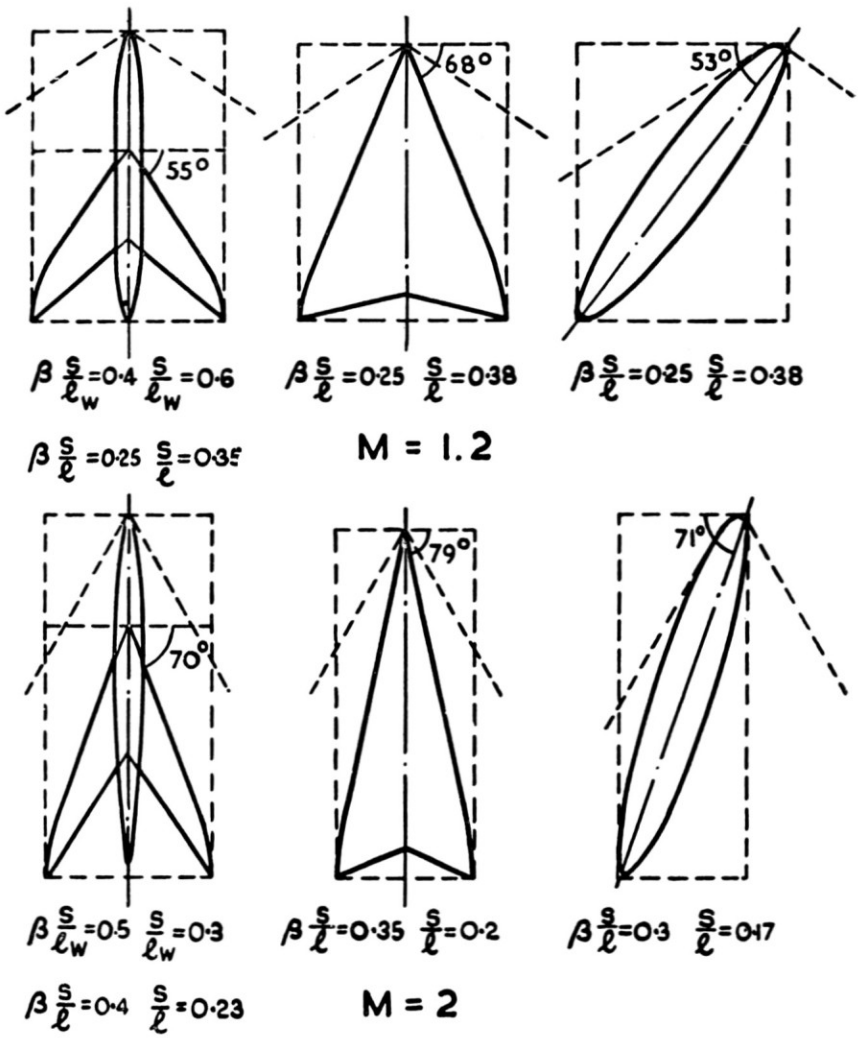

FIG. 3. Some typical aircraft shapes.

We do not propose to deal with a number of other layouts. Near-rectangular thin wings on a discrete fuselage are left out because their necessarily short lifting length must lead to high values of at least K_W so that their performance is altogether too poor, if the fuselage is non-lifting. If a fuselage with usually near-circular cross-sections is meant to lift, it must be ruled out as unsuitable for engineering applications because it is known that the separation lines cannot then be fixed so that the flow is essentially unsteady and liable to lead to asymmetries and the shedding of free vortices. Further, the flow over wing and fuselage is bound to change radically throughout the flight range. Similar reasons can be advanced against the use of other combinations of fuselages and thin wings with supersonic leading and/or trailing edges, such as some wings with planforms of delta, lozenge, or arrowhead shape. In some cases, flow

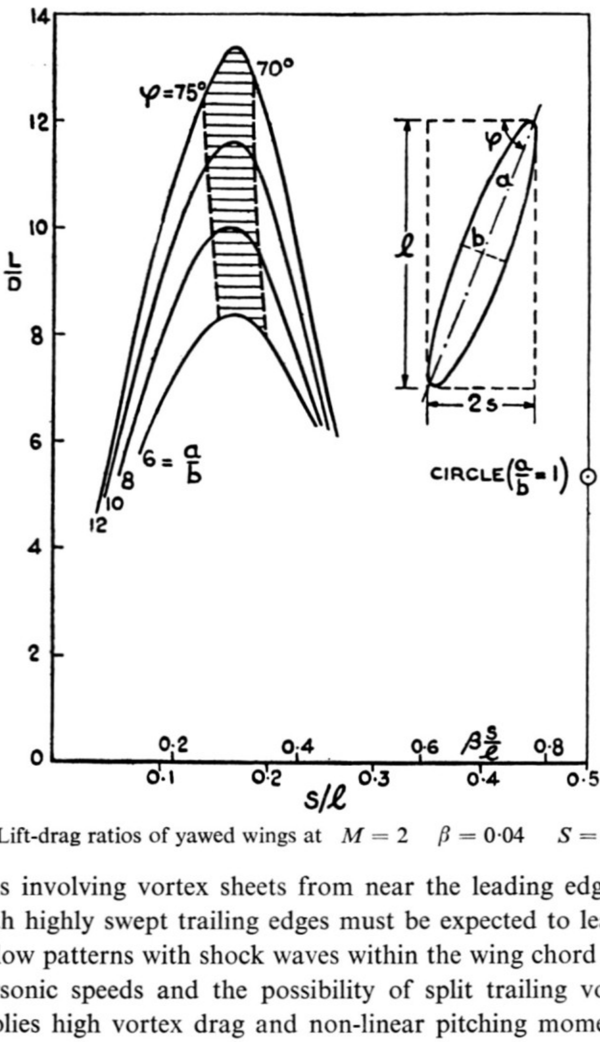

FIG. 4. Lift-drag ratios of yawed wings at $M = 2$ $\beta = 0{\cdot}04$ $S = 6000$ FT^2.

separations involving vortex sheets from near the leading edge in combination with highly swept trailing edges must be expected to lead to highly complex flow patterns with shock waves within the wing chord at transonic and supersonic speeds and the possibility of split trailing vortex sheets, which implies high vortex drag and non-linear pitching moments limiting the usable lift.

Sample properties of yawed wings which provide volume and lift simultaneously are shown in Fig. 4.* They have been determined by Smith[5]

* The values shown here are considerably lower than those of R. T. Jones[4] mainly because realistic values of the volume have been taken.

who extended earlier theories to cover thick wings. We find that the general level of L/D is of the order required from earlier performance considerations. We also find that the angle of yaw should lie within a fairly narrow band for best efficiency. The lift-drag ratio falls steeply if the angle of yaw is too high and the enclosing box too narrow ($\varphi = 90°$ where the curves end on the left-hand side), and again if the angle of sweep is too low and the box too wide (the mean spanwise direction is sonic where the curves end on the right-hand side). So this particular example confirms the earlier general statement that wings should lie well within the Mach cone from

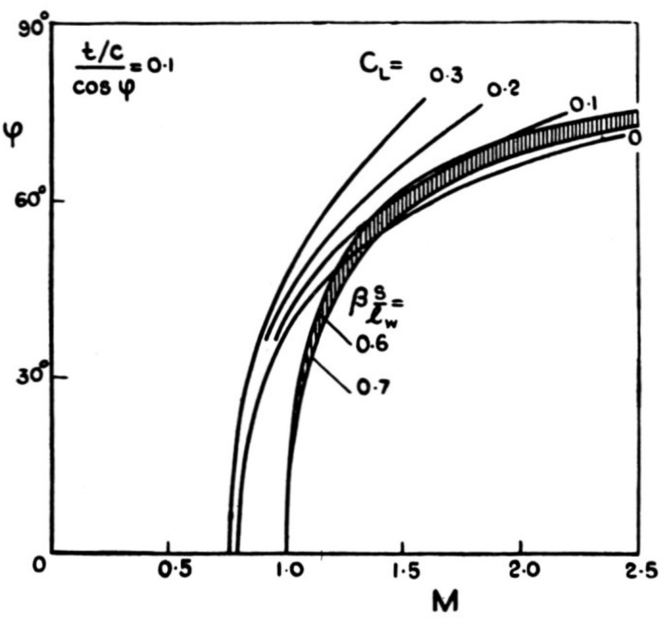

FIG. 5. Angles of sweep required for swept wings, according to Bagley (1956). (Sections with 30% roof-top pressure distributions.)

the apex. From the practical point of view, there is no need to concern ourselves with flows past wings with sonic or supersonic edges. A type of flow which could physically exist and be acceptable at the same time is the classical aerofoil flow with separation along the trailing edge only (Kutta condition). By reducing the angle of yaw at flight speeds below the cruise, this type of flow could be maintained, in principle, throughout the whole flight range. Its subsonic character can be maintained even at supersonic speeds because the Mach number component normal to the line of sweep is subsonic, of the order 0·7. However, there are further conditions to be satisfied in order to achieve this flow. R T. Jones[4] has already pointed out that the C_L must be limited but this is not all: the thickness of the wing and the details of the pressure distribution along the surface come into it, too. Bagley's work[6] is relevant here and we quote one of his results for wings of infinite span in Fig. 5, showing values of t/c, φ, C_L, and M which, in the right combination, can keep the flow just subcritical and of the classical type.

There are a great many other, as yet unsolved, difficulties to be overcome in order to maintain this type of flow, and failure to solve these would make results such as those in Fig. 4 meaningless and the whole scheme unacceptable in practice. Some are related to the design of the ends of the wing where the full sweep of the isobars must be maintained. Others are concerned with the stability and control of such wings and, by no means least of all, the economic utilization of the volume inside the wing for the purpose of accommodating passengers and other loads will present an exceedingly difficult task. At present, therefore, the yawed-wing aircraft would appear to be a very instructive example but not a really practical proposition.

A much more promising picture emerges when considering swept wing-fuselage combinations. This is the one case so far where it is reasonable to make a distinction between the length, l, of the volume-providing body or fuselage and the lifting length, l_W, of the wing: In the first place, the fuselage can be so designed as to carry no load except in the region of the wing; secondly, we know[7] that the thickness of the wing and fuselage together can be so designed, mainly by fuselage shaping, that the wave drag due to volume is, to a first order, that of the original fuselage alone. There is then in effect no wave drag which depends on the volume contained in the wing and on its shape.

Some typical results* are shown in Fig. 6. The drag factors chosen are: $K_0 = 1$, because one would attempt to approach this value of the best body of revolution of the given length and overall volume; and $K_V = K_W = 1$, because one would hope to find wing shapes with nearly elliptic loading both spanwise and lengthwise. The planform shape parameter, p_W, now refers to the planform of the wing along its surrounding box, and one would attempt to find shapes with $p_W < {}^1/_2$. The results in Fig. 6 are again of the order required from earlier performance considerations. They also confirm that the most efficient configurations lie well within the Mach cone from the nose, and the overall box size is again much the same as stated above. In addition, we find that the wing itself is also subsonic: There would seem to be no point in going to a sonic leading edge,

* To make the results roughly comparable with those for the other layouts, a fuselage of 180 ft length has been chosen with an overall volume parameter $\tau = 0{\cdot}06$; and a wing area of 3600 ft². $C_{DF} = 0{\cdot}006$ instead of about $0{\cdot}003$ because of the relatively larger wetted area.

say. (We may mention here that, for a sonic or supersonic leading edge, the values of the drag factors assumed are too good so that the actual L/D-values must be expected to be lower than those shown.) Near the maximum values of L/D, the Mach number component normal to the line of sweep is again around 0·7 and thus the classical aerofoil flow again offers itself as one which is both consistent with the assumption made in arriving at these results and also usable in engineering practice.

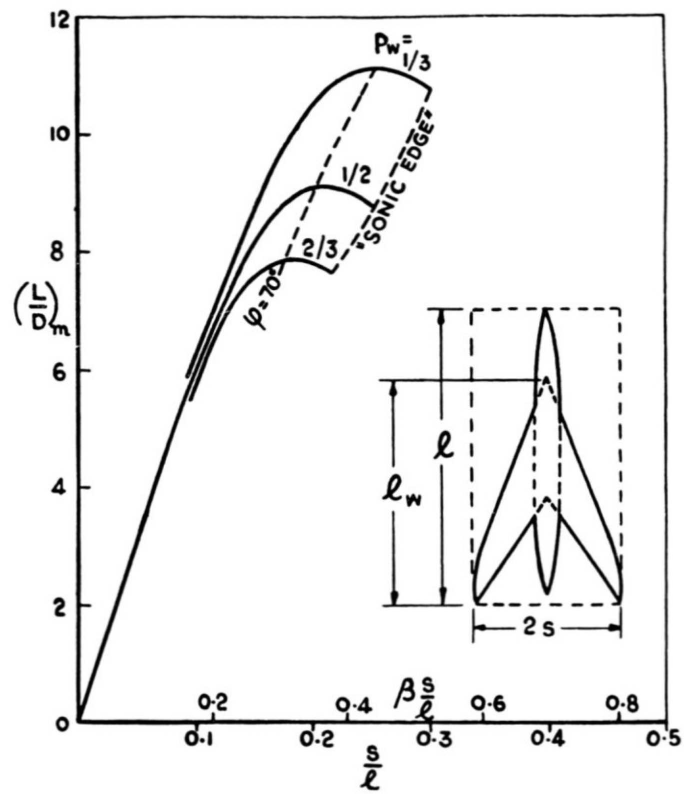

FIG. 6. Lift-drag ratios of swept wing-body combinations $M = 2$ $\beta = 0{\cdot}06$ $C_{df} = 0{\cdot}006$ $S = 3600\ FT^2$ $K_0 = 1$ $K_{wing} = 1$.

Having stipulated the general layout and the type of flow, the design aims are clear. The planform shape; camber and thickness distribution of the basic section which must again conform to Bagley's criterion, Fig. 5; and further, the spanwise variations of thickness, camber and twist; the shapes of the upper and lower wing-fuselage junctions; and the shapes of the fuselage cross-sections and of its own camber line can all be used to this end. The development of the three-dimensional boundary layer and the occurrence of shock waves on the wing appear to be the main stumbling blocks, tending to introduce additional separation lines. But we can now state with some confidence that these aims can be achieved, at least up to Mach numbers around 2 or 3. A great deal of background research in this field has now been done and is being summarised by Bagley[6]. The results of work on the aerodynamic design of sections, wings and bodies will be reported later at this Congress by Lock and Rogers[8], and Pearcey[9].

If the design for an efficient cruise performance appears feasible, it remains to consider the off-design conditions. Here, the obvious aim is to preserve what is anyway a subsonic type of flow at all speeds. This, however, is fraught with some difficulties which are the greater the higher the cruise Mach number. Not only does the angle of sweep increase with Mach number—from around 30° for $M = 0{\cdot}8$ to around 55° for $M = 1{\cdot}2$ to between 70° and 80° for $M = 2$—but also the whole shape is so radically affected by the cruise design as the Mach number goes up that the aircraft may no longer be fit to be flown at low speeds. To provide for a change of sweep angle with flight speed by means of "variable-sweep" schemes seems to have a great attraction for mechanical engineers but it must be borne in mind that sweep is only one shape parameter among many others which cannot be readily undone.

Nevertheless, designs up to cruise Mach numbers in the low supersonic range, up to 1·2 say, appear to be quite feasible all round and we have, therefore, come to regard the swept-winged aircraft as the logical extension of the classical aircraft into the low supersonic speed range, doing away with the "sound barrier" altogether. Aerodynamically, it has the same general layout, with separate means for providing volume, lift, and propulsion, and the same type of flow as the classical aircraft. Structurally, too, it does not radically differ from its predecessor and we might mention, for example, that the "structural thickness-chord ratio", $(t/c)/\cos\varphi$, and the "structural aspect ratio", A_s, determined from the actual length of span along the mean line of sweep and the mean chord normal to the line of sweep, may remain roughly constant and independent of flight Mach number and angle of sweep for aircraft with the same range. If t and $(t/c)/\cos\varphi$ are kept constant, then $A_s = A/\cos^2\varphi$. If also, as a rough rule, $M\cos\varphi = 0{\cdot}7$, then $A_s = 2AM^2$. Further, A is proportional to $1/M^2$ if $M.L/D$ is to remain constant, by equation (7), so that A_s does indeed not change to a first order.

This line of development is already actively pursued at the lower subsonic Mach numbers and aircraft exist, which have been designed on these lines known to us for some time[10]. There can be little doubt, that this will be pursued further in time. But we we have not yet discovered a really promising solution for higher supersonic speeds and even though Fig. 7

shows again in typical examples that the yawed wing and the swept wing-fuselage combination offer a satisfactory cruise performance on paper at $M = 2$, neither is really attractive at that or higher speeds. Now, Fig. 7 contains still another curve, for slender wings, with again the same performance, and it is these that we want to consider next.

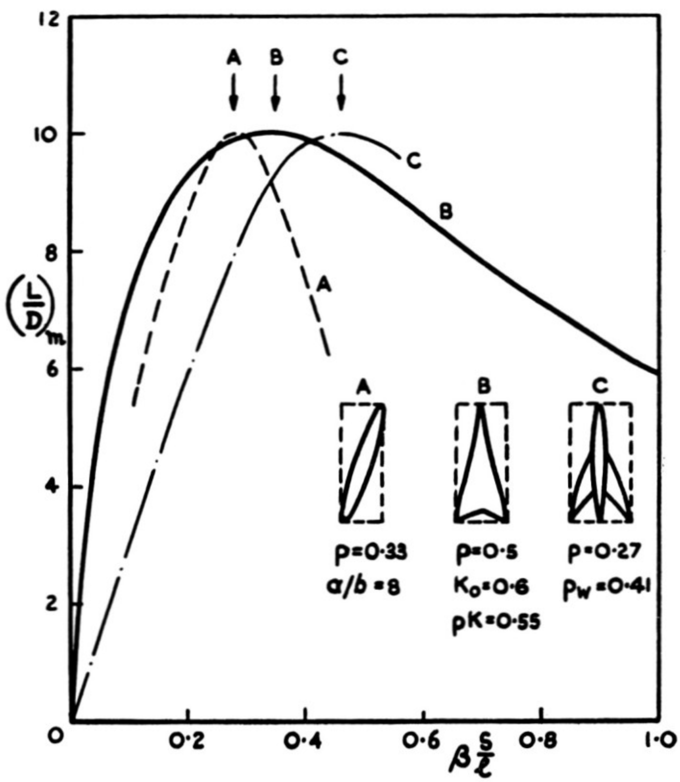

FIG. 7. Lift-drag ratios of three configurations at $M = 2$.

7. Slender Wings

A slender wing is a particularly simple shape to fit into the rather narrow boxes required at higher supersonic speeds. Slender wings are understood here to mean wings of near-triangular planform with subsonic leading edges and supersonic trailing edges. The leading edge is in general curved, with streamwise tips because this helps in designing for low drag factors. The trailing edge may be slightly swept. The value of the planform parameter p lies around $^1/_2$ and is not likely to be smaller than $^1/_3$ or greater than $^2/_3$. For the box sizes of interest, the wings may be regarded as both geometrically slender, i.e., $s/l \ll 1$, and aerodynamically slender, i.e., $\beta s/l \ll 1$. That the box sizes obtained earlier still apply is demonstrated in Fig. 8 where a wide variation of drag factors and planforms lead to only a small range of s/l, or $\beta s/l$, in which the highest L/D-values lie.

Now, the type of flow past these slender wings is the first to be fundamentally different from the classical aerofoil flow but it has the same general features which make it equally acceptable for engineering purposes.

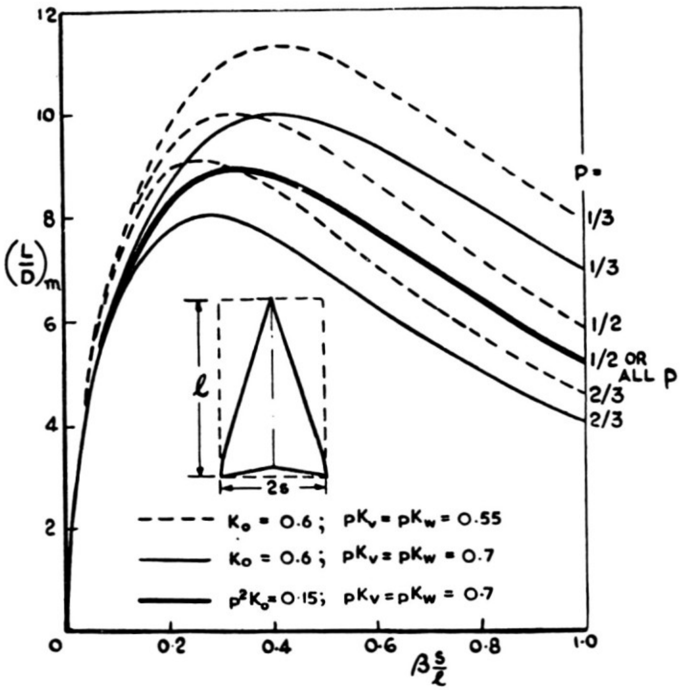

FIG. 8. Lift-drag ratios of slender wings at $M = 2$; $\beta = 1.73$ $\beta = 0{\cdot}04$ $C_{df} = 0.003$ $S = 6{\cdot}000\ FT^2$.

poses. Its main characteristics are the primary separation from all edges, leading to the formation of vortex sheets, at all flight conditions; and the absence of shock waves over the wing surface, at least under cruising conditions, as shocks are confined to a weak bow wave and another weak shock system behind the trailing edge, perturbations of the mainstream being genuinely small for once. As in the classical aerofoil flow, the external stream is predominantly inviscid and viscosity effects are confined to a thin layer with boundary-layer properties and to thin vortex sheets. The latter are above and close to the upper wing surface and constitute the main distinguishing feature. For the vortex sheets to be firmly anchored to the edges under all flight conditions, the edges must be geometrically and aerodynamically sharp. These flows have been found to be perfectly steady within the required flight range.

The main features of this flow have been first described by Roy[11] and Maskell[12] and Maskell and Weber[13] have considered it and the aerodynamic design principles involved in some detail. This type of flow must necessarily include one condition (i.e. one value of C_L and M), under which the leading edges are attachment lines. Above or below this C_L-value, the vortex sheets lie either wholly above or wholly below the wing. This occurs at $C_L = 0$ with symmetrical wings and at $C_L \neq 0$ with cambered, or warped, wings. To achieve this attachment at one C_L is one of the important design conditions. The flow then happens to be the same as that assumed in supersonic linearized theory, with trailing-edge separation only and a near-planar trailing vortex sheet. This appears to be one of the few cases in which this otherwise hypothetical flow has a real counterpart in nature. Thus a considerable body of theoretical work becomes available for the purpose of designing slender wings, notably the work of Ward and Lighthill [14,16], Weber[15] and R. T. Jones[3], Lord and Brebner[17] and others.

Some indication of the cruise performance of these wings is given in Fig. 8. The dashed lines are based on the assumption that the factor K_0 into the zero-lift wave drag can always be kept below unity, thus making use of Lighthill's theoretical prediction[14]. For simplicity of presentation the lift-dependent drag factors are taken to be equal and to have a value which must be considered as rather good ($K_V = K_W = 1{\cdot}1$ for $p = 1/2$). Good values of L/D, amply sufficient for long-range aircraft, are then achieved with wings in the whole p-range, although there remains an incentive to use wings with a low value of p. The full lines are based on the same assumption for K_0 but on less favourable assumptions for the lift-dependent drag, typical for wings where the camber is not very effective. Obviously, an efficient camber is worth striving for. Lastly, the thick line is obtained if K_0 is assumed to vary with p in such a way that it increases as p decreases, as an indication of increasing difficulties in the design. In this case, the distinction between wings of different p-values disappears and wings with lower p-values no longer offer an advantage. But even in this case, the performance is acceptable for long-range aircraft. With this potential performance, together with a perfectly usable type of flow, there is hardly any incentive left to try to make yawed or swept wings work at higher Mach numbers, as they do not offer a better performance. There is no incentive to use current conventional layouts, as their performance is certainly inferior.

There is unfortunately no time left to discuss the important and interesting low-speed aspects of slender wings in any detail (see e.g. Thwaites[2]). The main feature is, of course, that the coiled vortex sheets above the wing produce a non-linear lift force which may be considerably greater than the linear lift obtained on wings with trailing edge separation only. In practice, this non-linear lift makes a good take-off performance and low approach speeds possible, in spite of the low aspect ratio these wings have. How much the non-linear lift matters may be demonstrated by assuming the lift to be of the approximate form

$$C_L = \frac{\pi}{2} A\alpha + 2\alpha^2 = \pi \frac{s/l}{p}\alpha + 2\alpha^2 \tag{17}$$

and determining the wing area needed to sustain a given overall weight L at a given dynamic head q and angle of incidence α. This comes out as

$$S = \frac{L/q}{2\pi\alpha} \frac{1}{\frac{s/l}{2p} + \frac{\alpha}{\pi}} \tag{18}$$

where the term

$$\frac{L/q}{2\pi\alpha} = S_\infty \tag{19}$$

is the area needed with an unswept wing of infinite span. Values of S/S_∞ are shown in Fig. 9, where the upper curve indicates the increase in wing area, which is necessary if s/l is reduced in the case of linear lift only, whereas the lower curve includes the non-linear lift obtainable at $\alpha = \pi/10$, as an example. However, this curve still rises as a/l is decreased or the design Mach number increased, but it must be borne in mind that some increase in wing area, or decrease in wing loading, will be needed anyway because the faster aircraft will also fly higher. Since, further, the decrease in s/l with M may well be less than indicated in Fig. 8, there is no undue discrepancy between low-speed and cruise performances on this count alone. There are, however, other problems, notably in the field of flight dynamics, which we have no time to discuss here.

We turn now to a brief discussion of some of the aerodynamic problems which are typical of slender wings. The shedding of vorticity from side edges presents a new kind of problem altogether. Early work in this field is discussed by Thwaites(2), and Maskell(18) has established similarity laws for the initial rate of shedding of vorticity for conical bodies as it depends on the angles of incidence and sweep, on the edge angle and its droop, and on the cross-sectional shape. The first of these—that similar flows are obtained when $\alpha/(s/l)$ is the same—is already well-known. Observations of non-conical flow patterns, including some which lead to an asymmetric shedding of vorticity, have been described by Maltby(19). The results of this work have a bearing on a variety of aspects: on flight dynamics, on forces and moments, and also on vortex drag. We note, for example, that $K_V = 1$ can no longer be regarded as a lower bound, as it relates to the minimum vortex drag for wings of given overall lift and span, which leave a plane sheet of trailing vortices behind them, whereas the shape of the sheet behind slender wings is essentially non-planar.

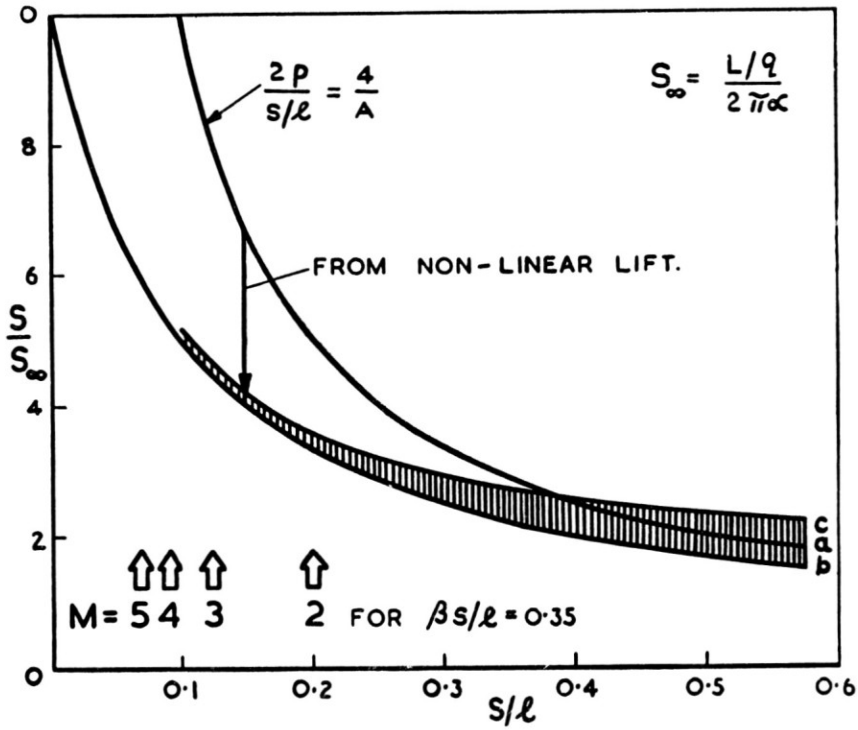

FIG. 9. Wing areas required for low-speed performance. P = 1/2. *a* : R. T. Jones lift. *b*: with non-linear lift at $\alpha = \pi/10$. *c*: including practical allowance.

Another set of problems is concerned with the development of the boundary layer which is essentially three-dimensional. New methods for calculating these must be devised, and the present state of our knowledge has been summarized by Cooke and Hall[(20)]. Further, new criteria must be found to define what constitutes pressure fields which are either favourable or unfavourable to separation occurring somewhere on the surface, apart from the separation from the edges. Such secondary separations are obviously undesirable. The relevant criteria have been established by Maskel and Weber[(13)].

Further problems arise when considering the wave drag at zero lift for thick wings. Lighthill's prediction[(14)] that the wave drag can be reduced below that of the corresponding body of revolution by spreading the volume spanwise has been followed up and typical results, by Weber[16], are shown in Fig. 10. Depending on the slope and the curvature at the trailing edge of a family of area distributions, the drag factor K_0 is found to be bounded below by an envelope. Then two further problems arise: Firstly, the reliability of such results, as some of the shapes implied may cease to be

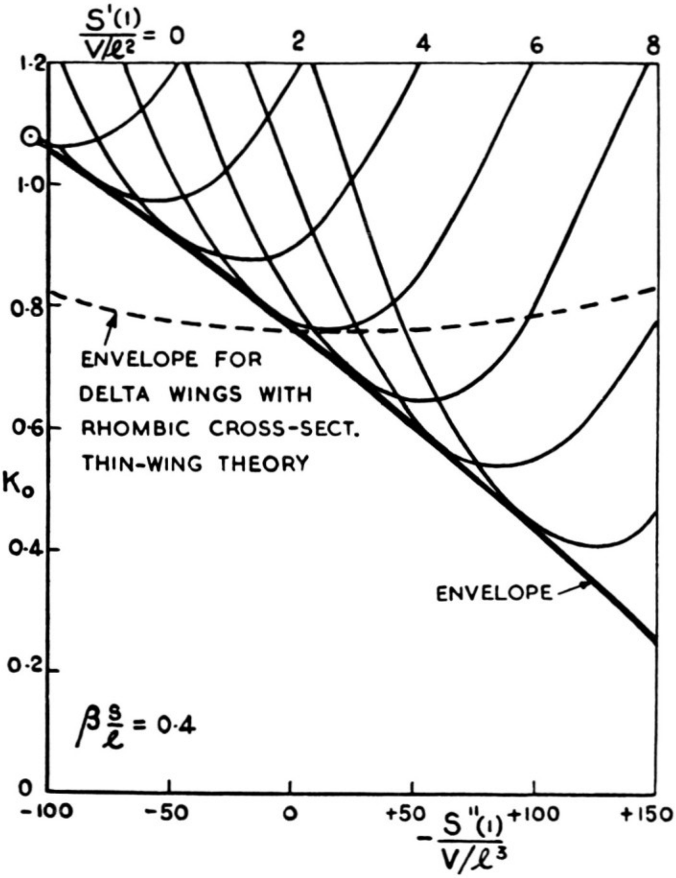

FIG. 10. Zero-lift wave drag according to slender wing theory for a family of wings.

$$\frac{S(x)}{V/l} = (1-x)x^2 \sum_0^3 a_n x^n, \quad k = 1{\cdot}85.$$

slender and smooth, and as different theories may give different numerical answers. Secondly, the question of how low the drag factor can be designed to be, as numerical results depend on the particular family of shapes chosen. Thus optimization procedures appear in a new light and some of these, such as some popular "area rules", become suspect.

Similar problems occur with regard to the wave drag due to lift and, as an illustration, Fig. 11 shows some results from an extension of Adams and Sears[21] not-so-slender wing theory, due to Weber, for the drag factor K_W for a family $L(x)$ of loadings along the chord of the wing, which have different values of the load $L(1)$ at the trailing edge and different positions $x_{c.p.}$ of the centre of pressure. Again, the curves are bounded below by an envelope and some of the values of K_W along this envelope are well

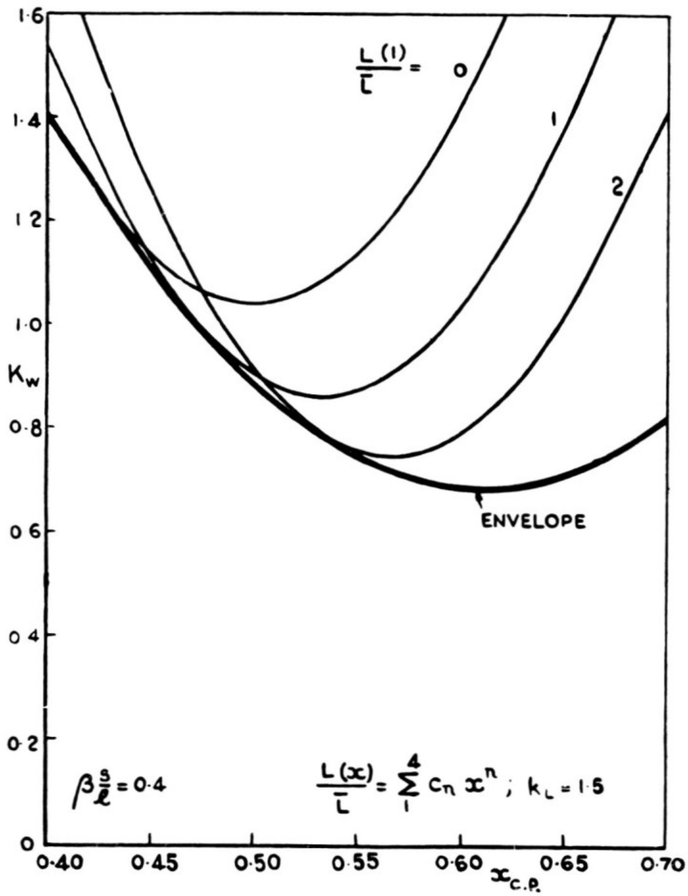

FIG. 11. Lift-dependent wave drag according to not-so-slender wing theory for a family of wings.

below R. T. Jones[3] "lower bound". This kind of result has some bearing not only on the drag itself but also on the trim problem. Evidently, fore-planes which invariably introduce a trim-drag penalty and considerable complications in the flow pattern and flight behaviour are no longer needed with properly designed slender wings.

To end this discussion of slender wings, we might look at some of the experimental results obtained. Fig. 12 gives measured values of the zero-lift wave drag for a number of wings of different thickness and semi-span-length ratio, tested in wind tunnels and in free flight. The area distribution is always the same (designated "Lord V"); it is that used in some of the calculations given earlier, and the experiment is seen to confirm the assumptions made. It was possible to design these wings to lead to genuinely small perturbations of the mainstream, in spite of sometimes large values of the volume parameter, and to have wholly favourable pres-

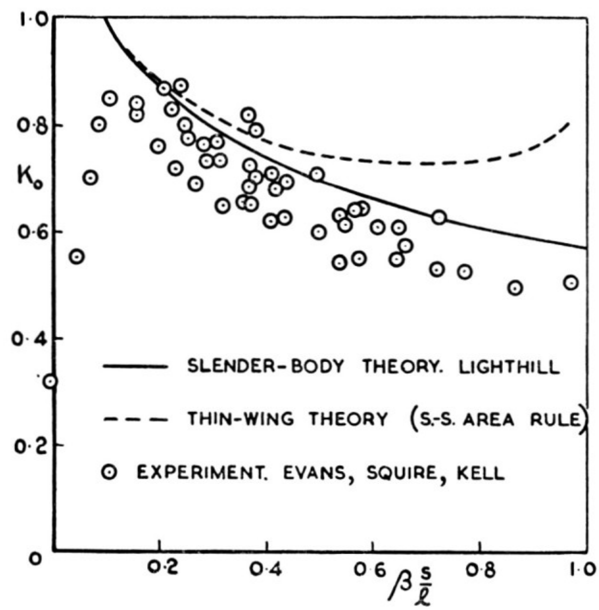

FIG. 12. Volume-dependent wave drag for 8 wings.

sure fields. In view of these results, one would call configurations designed to conform to the conventional area rule ($K_0 = 1$) wholly unsatisfactory.

Figure 13 gives measured values of the lift-dependent drag for a number of wings, where K is defined in the usual way as

$$K = 2\pi \frac{s/l}{p} \cdot \frac{C_D - C_{D0}}{C_L^2} \tag{20}$$

so that the points at $\beta s/l = 0$ give the values of K_V and the slope of the lines the values of K_W. It will be seen that the values assumed earlier can indeed be realized and that the drag is considerably less than what would have been obtained had there been linear lift only and no suction force. It is of interest to note, that the shape of this particular planform is very close to the von Kármán ogive and,therefore, the one which R. T. Jones[3] has shown to give elliptic span—and chord—loadings ($K_V = K_W = 1$). Now even though the flow assumed in that theory cannot be realized in practice, other flows with almost the same drag values can, and that in a variety of ways. In the first place, one model tested was cambered to have attachment along the leading edge at $C_L = 0{\cdot}1$; linear lift was then obtained and "full suction" as assumed in the theory. Secondly, another model was uncambered so that, at $C_L = 0{\cdot}1$, vortex sheets were

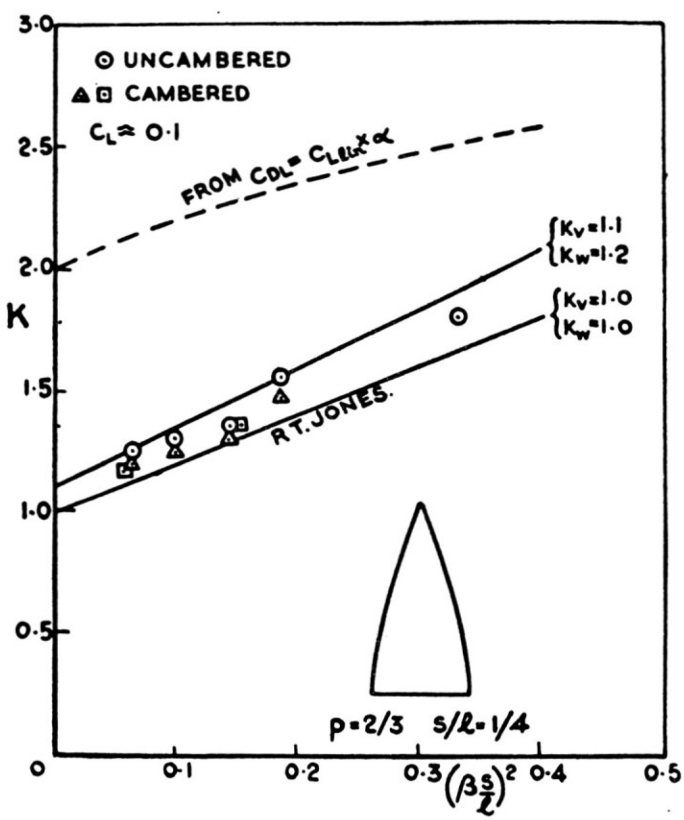

FIG. 13. Lift-dependent drag factor from experiments by Squire and Evans.

fully developed; non-linear lift was then obtained together with some suction below the vortex sheets, acting on forward-facing surfaces of the thick wing. The third model was cambered to have attachment at $C_L = 0{\cdot}05$ so that at $C_L = 0{\cdot}1$ some, weaker, vortex sheets had developed. That the drag is the same in all cases is, of course, related to the particular planform and thickness used and need not be repeatable with other wings.

8. *Conclusions*

In conclusion, we may say that it has been shown to be worthwhile to bear in mind the practical application at an early stage. If the task is the simplest, namely, to achieve a given range, then even the crudest knowledge of the structural, propulsive, and aerodynamic components of an aircraft is sufficient to define some overall dimension into which the aircraft must fit. We have seen that, in the case of the classical aircraft, this leads to the conclusion that we need to concern ourselves only with wings of moderate or large aspect ratios. In the case of supersonic flight, with different means of propulsion and a new set of aerodynamics, this leads to the definition of box sizes with certain relations between span

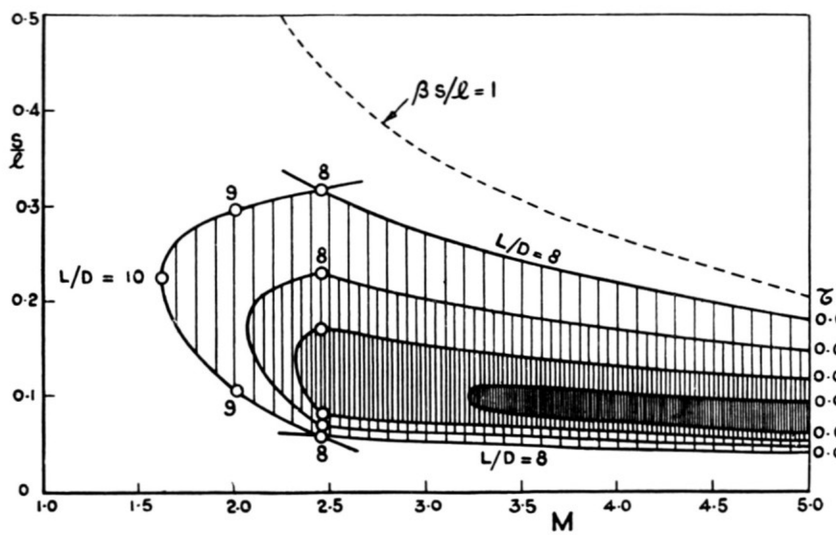

FIG. 14. Semispan-length ratios for given values of L/D. p = 1/2, $S = 6000\ FT^2$ $K_1 = K_w = 1$ K_0 for Lord V.

and length, into which the aircraft must fit. These boxes are always longer than wide and become narrower as the design Mach number goes up so that the aircraft lies always well within the Mach cone from its nose.

The next important step is to find shapes to fit into these boxes with a suitable type of flow which not only leads to the required performance but is also acceptable for engineering purposes in that it is a real flow, steady, and preferably the same throughout the flight range. We have recalled that the classical aerofoil flow suits these conditions perfectly but that it is also limited to just the classical aircraft layout and to essentially subsonic flight speeds. We have then seen that the regime of this flow and layout can be extended to low supersonic flight speeds by making use of sweep. And, finally, we have discussed an entirely new type of flow, that past slender wings, and shown that this leads to a natural solution for supersonic aircraft.

As we have been mainly concerned with examples at lower supersonic speeds and at Mach numbers around 2, we would end with a last Fig. 14, which gives box sizes and expected aerodynamic performances for slender-wing layouts up to higher Mach numbers. Lines along which $L/D = 8$ for $\tau = 0{\cdot}04$ are closed at the lower end by a curve along which L/D varies in a manner which is typically needed to balance the loss of propulsive efficiency of turbojet engines as the Mach number decreases. Within this band lie all the wings which give a better performance or allow a greater volume. Such a diagram gives not a few pointers to future developments and makes it clear at the same time that the aerodynamic design of supersonic aircraft is, as always, as much a matter of low-speed aerodynamics as of high-speed aerodynamics. But whereas we can see real aircraft emerge now at the lower Mach numbers, the higher speeds will demand not only the integration of volume and lifting surface but also of propulsion. We also realize that research into fluid dynamics and chemical kinetics must go together with that into structures, materials and systems. There can be little doubt, however, that human flight will in time far exceed its present limitations.

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DISCUSSION

G. H. LEE: Dr. Küchemann has referred to the possibilities of the yawed aerofoil as a configuration for supersonic flight. My Company has given some consideration to this idea recently and I would like to present to you, briefly, some of the results we have obtained, for I think that an International Conference such as this is a suitable occasion for discussing new aircraft concepts, even though, as in this case, the idea may seem bizarre at first sight; it was, after all, at the first I.C.A.S. in Madrid that R. T. Jones introduced the yawed wing idea to many of us for the first time.

As we see it, the yawed wing aeroplane consists of a wing large enough to contain the passengers within the basic aerofoil contour; this means that it must be a fairly large aeroplane, probably at least 300 ft. from tip to tip. The crew would be housed in a nacelle (or small fuselage) at the forward tip and there would be a fin at the rearward tip to give directional stability. Control would be obtained by means of "ailerons" (capable of moving symmetrically or anti-symmetrically as required) and a rudder. The engines would be mounted near the centre of the trailing edge.

The yawed wing is not only an efficient aeroplane when cruising supersonically, but should be thought of as the ideal solution to the problem of variable geometry since the angle of yaw can be changed in flight, the main loads being supported on a perfect "air bearing" with only secondary loads having to be taken through mechanical bearings; for to change the angle of yaw, it will be necessary only to change the angle of the fin, to rotate the crew cabin and to alter the angle of the jet issuing from the engines by means of some sort of adjustable propelling nozzle. This should be compared with "conventional" variable geometry aeroplanes in which major loads have to be taken through mechanical joints, with consequent problems of design, extra weight, and maintenance. The yawed wing indeed poses some aerodynamic problems, but once these have been overcome, the solution is permanent; the air does not need maintenance.

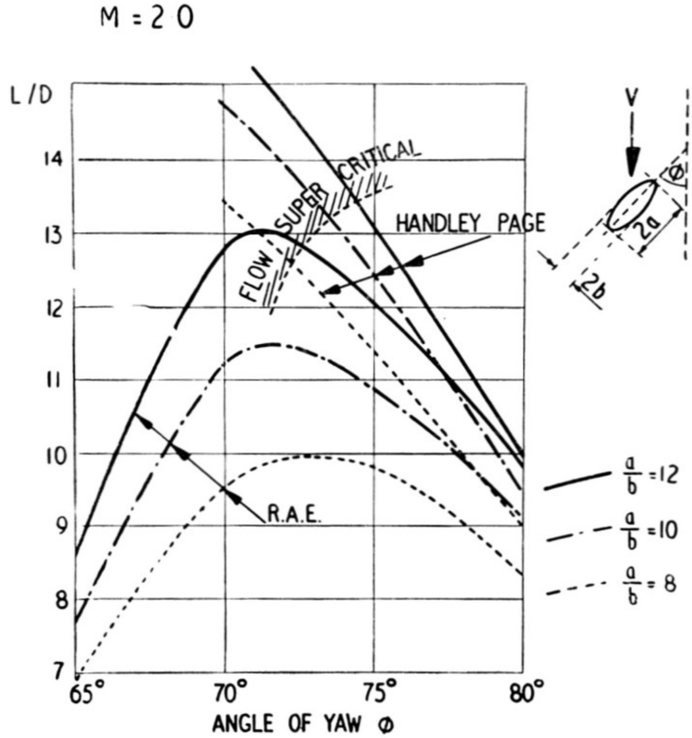

FIG. A1.

To deal first with the basic cruising performance, we have estimated, at $M=2$, the performance of the wing already considered by Küchemann (see his Fig. 4) by regarding it as sufficiently yawed to be sub-critical in the flow normal to the length of the wing, it being assumed that at the forward tip the crew nacelle is waisted and at the rear tip a body associated with the fin is similarly treated so as to maintain the sweep of the isobars for the length of the wing; in this way the wing is shock free, the shock waves being associated with the two tip bodies, for which a drag allowance was made. Using this method, the results shown in the first figure were obtained. It will be see that agreement with the RAE estimates is fair and it may be concluded that for the Mach number in question, namely 2.0, a ration of $L/D = 11.0$ may be expected for a yawed wing of practical proportions. (Note the shaded line indicating angles of yaw below which the normal Mach number will exceed the critical.) This figure, $L/D = 11.0$, may be compared with the typical value $L/D = 9.0$ for a slender wing at $M=2.0$.

In further support of the above, we have tested a yawed wing at low speed in a wind tunnel, and for angles of yaw of about 70° obtained a low induced drag, K_V of Küchemann's paper being close to unity.

However, a supersonic aeroplane spends much of its time flying non-supersonically, (for take-off and landing, climb, stand-off, etc) and it is in such phases that variable geometry pays. Subsonically, the yawed aircraft flies best at $M = 0.34$, with 30° of yaw and with $L/D = 24$, far higher than a slender wing under comparable conditions.

Taking, as an example, an aeroplane of 350,000 lbs. A.U.W. capable of transatlantic flight at $M = 2.0$ and it will be found that the payload (of passengers) is 24,000 lb (120 passengers) for the slender wing arrangement. The improvement that would accrue to the comparable yawed wing aeroplane, on account of better subsonic performance alone, is an increase of 50% in the payload, namely another 12,000 lbs.

M = 2·0

TYPE	SLENDER WING	YAWED WING
ALL-UP WEIGHT	350,000 LB.	350,000 LB.
B.E.W.	176,000 LB.	176,000 LB.
CRUISE FUEL	100,000 LB.	86,000 LB.
OTHER FUEL	50,000 LB.	40,000 LB.
SAVING IN FUEL	——	24,000 LB.
PAYLOAD	24,000 LB.	48,000 LB.
ACTUAL CRUISE L/D	9·0	11·0
EFFECTIVE CRUISE L/D	9·0	11·5

FIG. A2.

The full comparison* is shown in Fig. A2, from which it is seen that, at the best, the yawed wing can carry twice the payload of the conventional aeroplane; even accepting Küchemann's statement that the two types have the same supersonic cruising performance (both having $L/D = 10$, see his Fig. 7), the better off-design performance of the yawed wing still gives 50% more payload, a worth while increase. (N.B. In Fig. A2. "Effective L/D" allows for the better off design performance of the yawed wing and should be used in comparisons based on the Breguet range parameter $\frac{L}{D} \cdot \frac{M}{C}\Big)$.

At $M = 5.0$, RAE estimates indicated no significant difference in cruising L/D between the slender and yawed wings (about $7^1/_2$ in both cases) but because the optimum slender wing has an aspect ratio of only 1/3rd at this Mach number, such a wing would be poor in the climb or during stand-off and would need either variable geometry or V.T.O.; hence the yawed wing will gain greatly from its good subsonic performance at small angles of yaw when considered for operation at the higher Mach numbers.

To conclude, the yawed wing concept looks strange and will present many problems for solution, but it seems to offer the prospect of valuable performance gains.

* Assuming equal Basic Equipped Weights, a reasonable assumption since both wings are of fairly simple geometry, while the smaller engines of the yawed wing will offset such variable geometry as it has.